

# Graph Random Neural Network for Semi-Supervised Learning on Graphs

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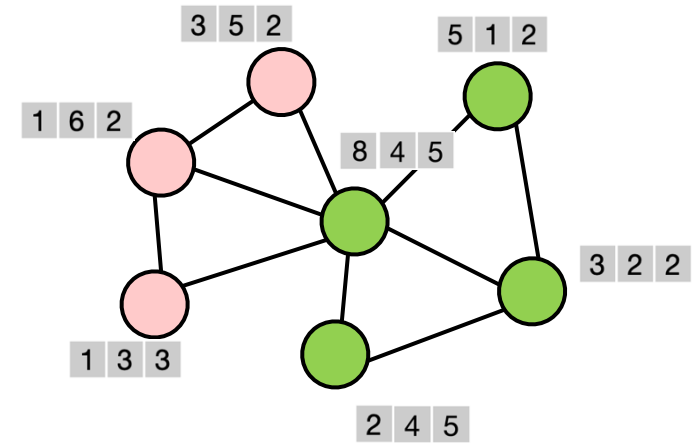
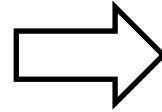
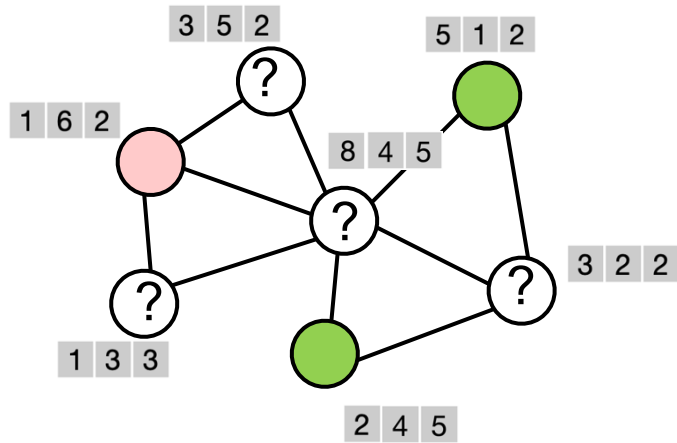


Microsoft



**BOSCH**  
Invented for life

# Semi-Supervised Learning on Graphs

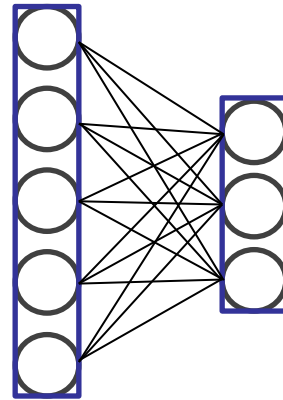
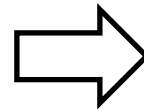
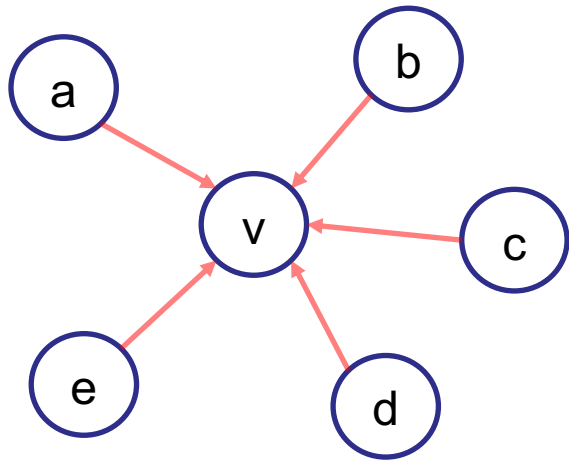


**Input:** a partially labeled & attributed graph

**Output:** infer the labels of unlabeled nodes

# Graph Neural Network (GNN)

Graph Convolution Network:



node  $v$ 's embedding at  $k + 1$

non-linear activation function (e.g. ReLU)

$$\mathbf{H}^{k+1} = \sigma(\hat{\mathbf{A}}\mathbf{H}^{(k)}\mathbf{W}^{(k)})$$

normalized Laplacian matrix

$$\mathbf{H}^{k+1} = \sigma\left(\mathbf{W}^k \sum_{u \in N(v) \cup v} \frac{\mathbf{H}_u^k}{\sqrt{|N(u)||N(v)|}}\right)$$

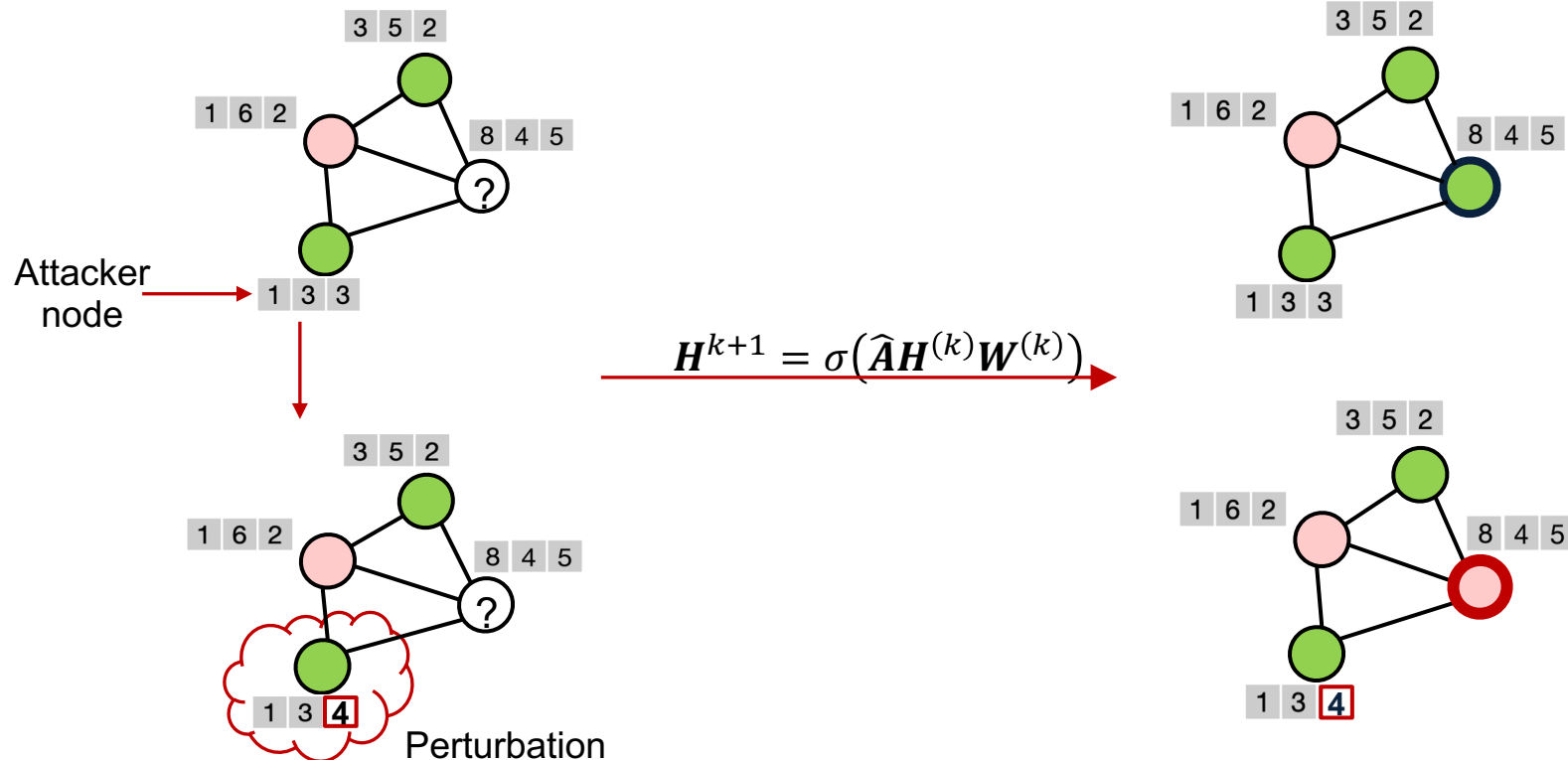
the neighbors of node  $v$

# Graph Neural Networks

$$\mathbf{H}^{k+1} = \sigma(\widehat{\mathbf{A}}\mathbf{H}^{(k)}\mathbf{W}^{(k)})$$

a deterministic propagation

1. Each node is highly dependent with its neighborhoods, making GNNs **non-robust** to noises



# Graph Neural Networks

$$\mathbf{H}^{k+1} = \sigma(\widehat{\mathbf{A}}\mathbf{H}^{(k)}\mathbf{W}^{(k)})$$

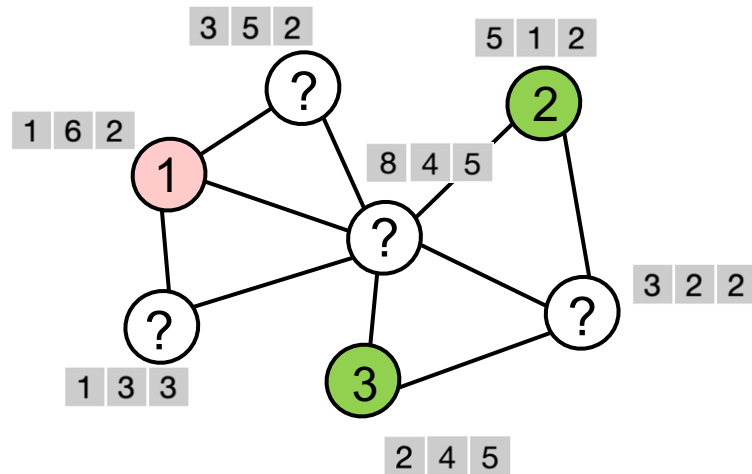
feature propagation is  
Laplacian smoothing,  
coupled with  
non-linear transformation

1. Each node is highly dependent with its neighborhoods, making GNNs **non-robust** to noises
2. Stacking many GNNs layers may cause **over-smoothing**.

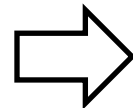
# Graph Neural Networks

1. Each node is highly dependent with its neighborhoods, making GNNs **non-robust** to noises
2. Stacking many GNNs layers may cause **over-smoothing**.
3. Under semi-supervised setting, standard training method is easy to **over-fit** the scarce label information.

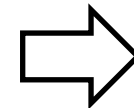
*Standard training method for GNN:*



$$\mathbf{H}^{k+1} = \sigma(\hat{\mathbf{A}}\mathbf{H}^{(k)}\mathbf{W}^{(k)})$$



GNN

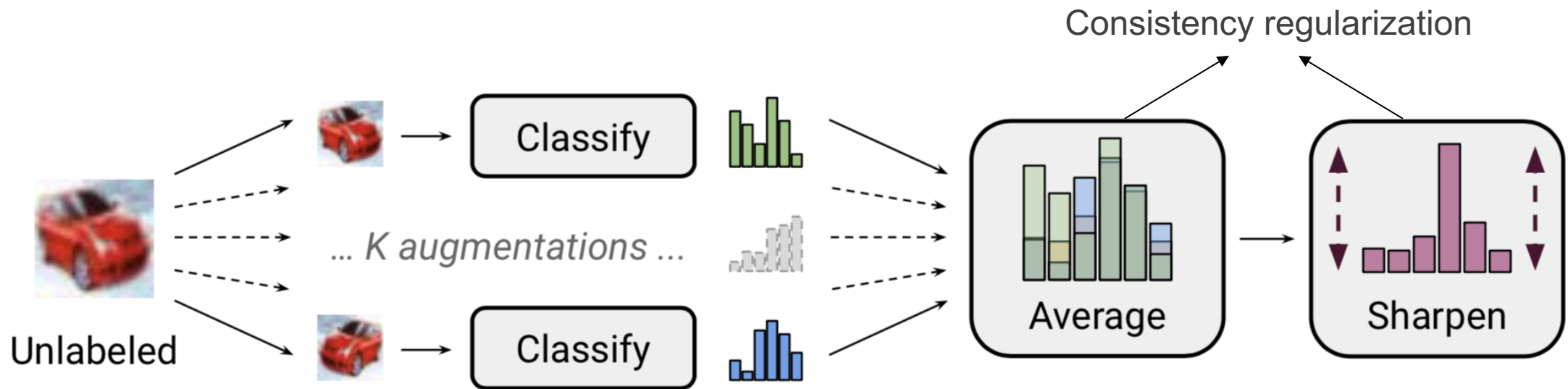


Loss function:  
 $\mathbf{y}_1^T \log(\hat{\mathbf{y}}_1) + \mathbf{y}_2^T \log(\hat{\mathbf{y}}_2) + \mathbf{y}_3^T \log(\hat{\mathbf{y}}_3)$

Cannot fully leverage  
unlabeled data

# Recent advances in Semi-Supervised Image Classification

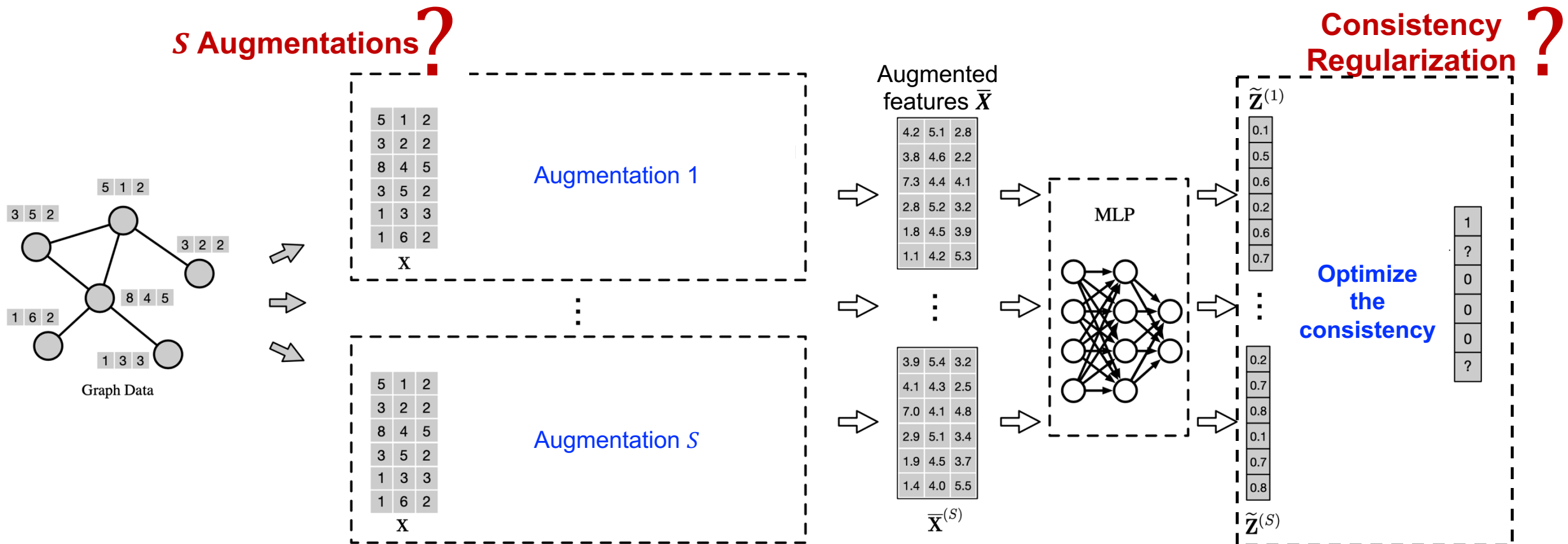
- Improving models' generalization through image data augmentation and consistency regularization.



*(Picture from MixMatch's paper)*

# Graph Random Neural Network (GRAND)

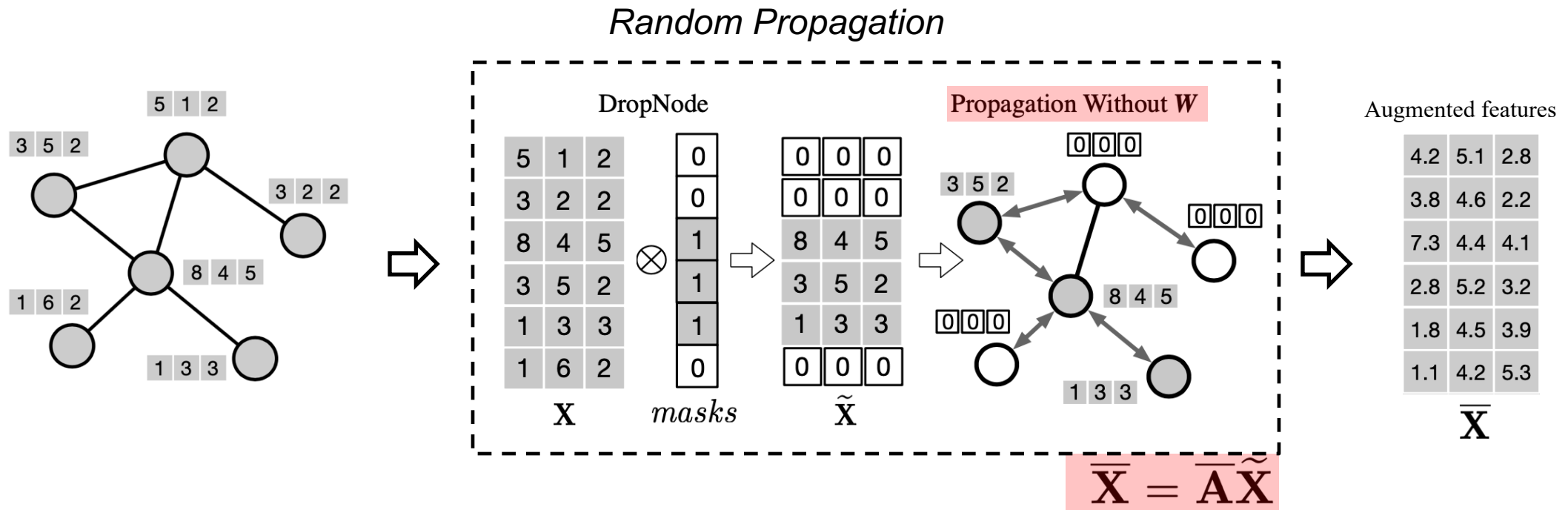
- Consistency Regularized Training:
  - Generates  $S$  data augmentations of the graph
  - Optimizing the consistency among  $S$  augmentations of the graph.





# Graph Random Neural Network (GRAND)

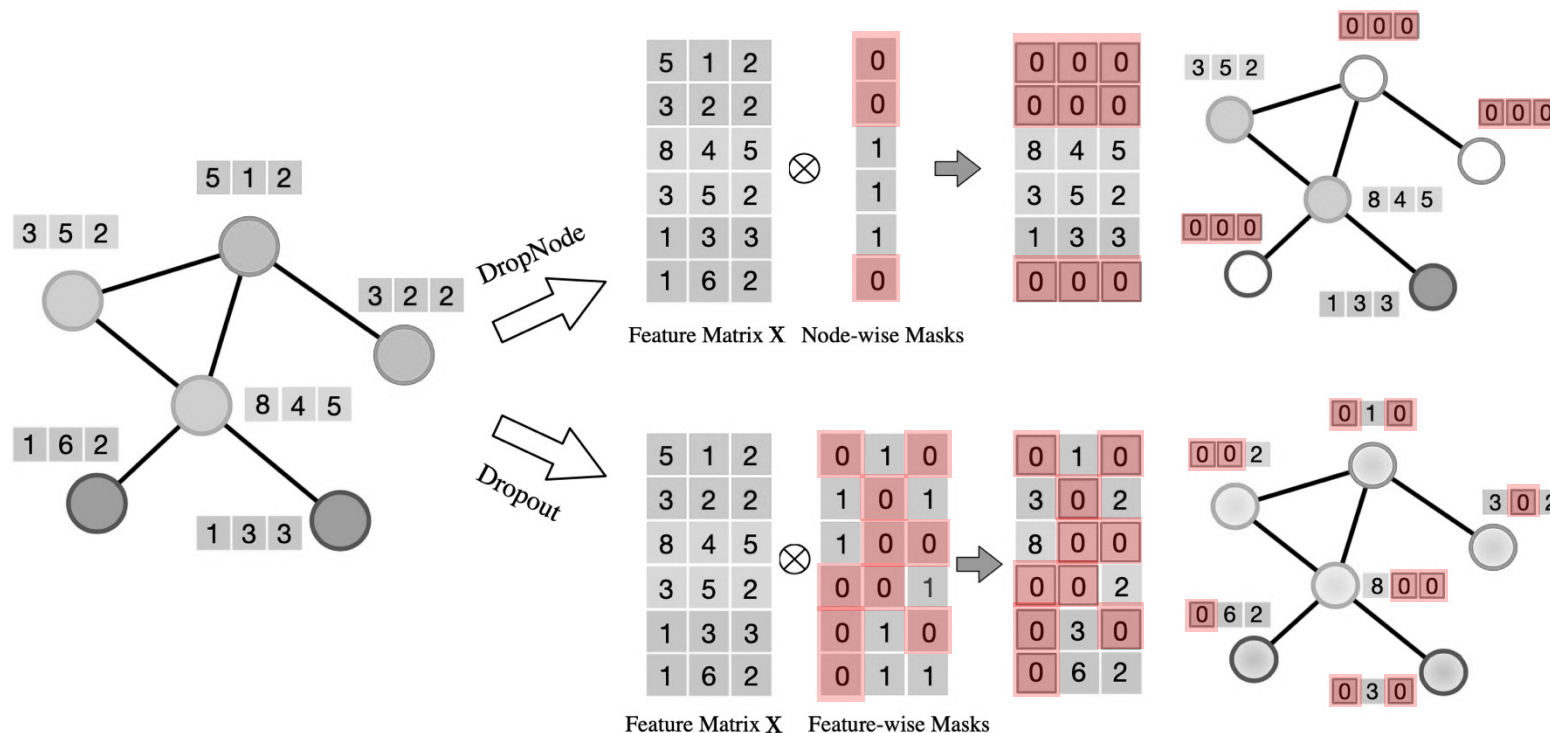
- **Random Propagation** (DropNode + Propagation):
  - **Enhancing robustness**: Each node is enabled to be not sensitive to specific neighborhoods.
  - **Mitigating over-smoothing and overfitting**: Decouple feature propagation from feature transformation.



- Feng et al. Graph Random Neural Networks for Semi-Supervised Learning on Graphs. <https://arxiv.org/abs/2005.11079>, 2020
- **Code & data** for Grand: <https://github.com/Grand20/grand>

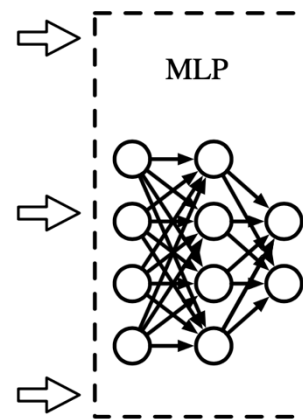
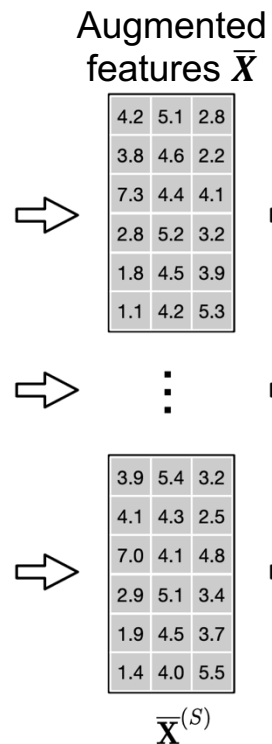
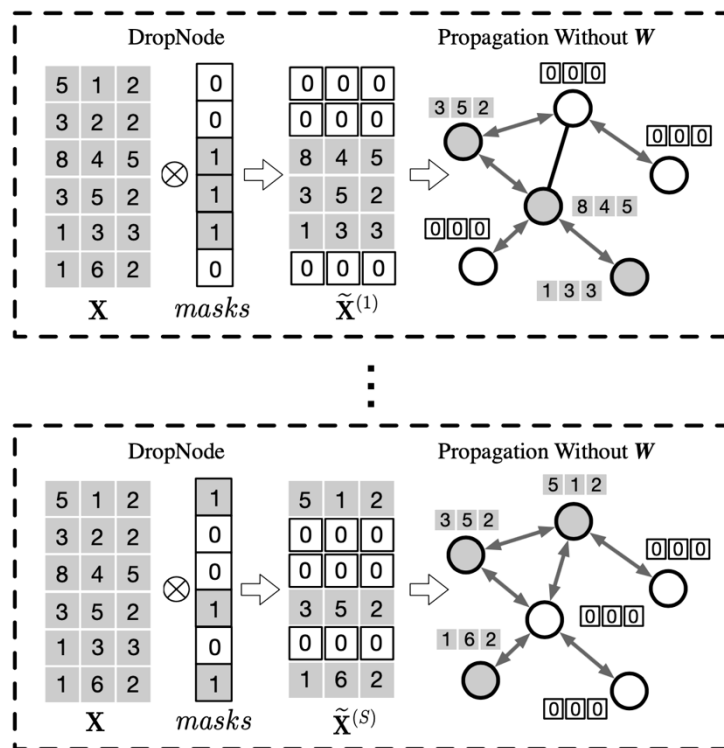
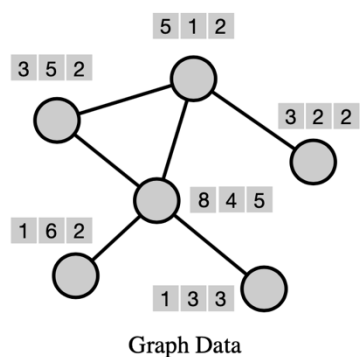
# Random propagation: DropNode vs Dropout

- Dropout drops each element in  $X$  independently
- DropNode drops the entire features of selected nodes, i.e., the row vectors of  $X$ , randomly

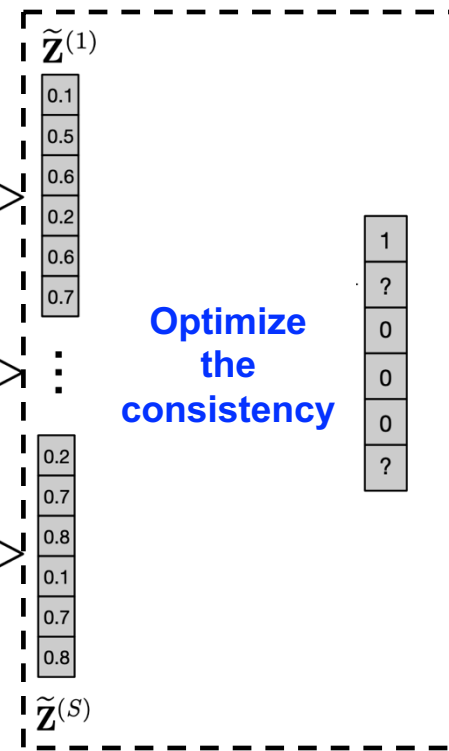


# Graph Random Neural Network (GRAND)

## S Augmentations



## Consistency Regularization ?

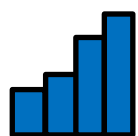
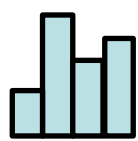


Random Propagation as data augmentation

- Feng et al. Graph Random Neural Networks for Semi-Supervised Learning on Graphs. <https://arxiv.org/abs/2005.11079>, 2020
- Code & data for Grand: <https://github.com/Grand20/grand>

# GRAND: Consistency Regularization

Distributions of a node  
after augmentations



Average



$$\bar{\mathbf{Z}}_i = \frac{1}{S} \sum_{s=1}^S \tilde{\mathbf{Z}}_i^{(s)}$$

$$\mathcal{L}_{sup} = -\frac{1}{S} \sum_{s=1}^S \sum_{i=0}^{m-1} \mathbf{Y}_i^\top \log \tilde{\mathbf{Z}}_i^{(s)}$$

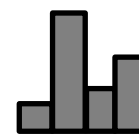
+

⇒

$$\mathcal{L} = \mathcal{L}_{sup} + \lambda \mathcal{L}_{con}$$

$$\mathcal{L}_{con} = \frac{1}{S} \sum_{s=1}^S \sum_{i=0}^{n-1} \mathcal{D}(\bar{\mathbf{Z}}'_i, \tilde{\mathbf{Z}}_i^{(s)})$$

Sharpening



$$\bar{\mathbf{Z}}'_{ik} = \bar{\mathbf{Z}}_{ik}^{\frac{1}{T}} \Bigg/ \sum_{j=0}^{C-1} \bar{\mathbf{Z}}_{ij}^{\frac{1}{T}}$$

# Graph Random Neural Networks (GRAND)

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**Input:**

Adjacency matrix  $\hat{\mathbf{A}}$ , feature matrix  $\mathbf{X} \in \mathbb{R}^{n \times d}$ , times of augmentations in each epoch  $S$ , DropNode probability  $\delta$ .

**Output:**

Prediction  $\mathbf{Z}$ .

1: **while** not convergence **do**

2:   **for**  $s = 1 : S$  **do**

3:     Apply DropNode via Algorithm 1:  $\tilde{\mathbf{X}}^{(s)} \sim \text{DropNode}(\mathbf{X}, \delta)$ .

4:     Perform propagation:  $\bar{\mathbf{X}}^{(s)} = \frac{1}{K+1} \sum_{k=0}^K \hat{\mathbf{A}}^k \tilde{\mathbf{X}}^{(s)}$ .

5:     Predict class distribution using MLP:  $\tilde{\mathbf{Z}}^{(s)} = P(\mathbf{Y} | \bar{\mathbf{X}}^{(s)}; \Theta)$ .

6:   **end for**

7:   Compute supervised classification loss  $\mathcal{L}_{sup}$  via Eq. 4 and consistency regularization loss via Eq. 6.

8:   Update the parameters  $\Theta$  by gradients descending:

$$\nabla_{\Theta} \mathcal{L}_{sup} + \lambda \mathcal{L}_{con}$$

9: **end while**

10: Output prediction  $\mathbf{Z}$  via Eq. 8.

---

**Generate  
 $S$  Augmentations**

**Consistency  
Regularization**

## Consistency Regularized Training Algorithm

- Feng et al. Graph Random Neural Networks for Semi-Supervised Learning on Graphs. <https://arxiv.org/abs/2005.11079>, 2020
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# Graph Random Neural Network (GRAND)

- With Consistency Regularization Loss:

- Random propagation can enforce the consistency of the classification confidence between each node and its all multi-hop neighborhoods.

$$\mathbb{E}_\epsilon (\mathcal{L}_{con}) \approx \mathcal{R}^c(\mathbf{W}) = \sum_{i=0}^{n-1} z_i^2 (1 - z_i)^2 \text{Var}_\epsilon (\overline{\mathbf{A}}_i \tilde{\mathbf{X}} \cdot \mathbf{W})$$

$$\mathcal{R}_{DN}^c(\mathbf{W}) = \frac{\delta}{1 - \delta} \sum_{j=0}^{n-1} \left[ (\mathbf{X}_j \cdot \mathbf{W})^2 \sum_{i=0}^{n-1} (\overline{\mathbf{A}}_{ij})^2 z_i^2 (1 - z_i)^2 \right]$$

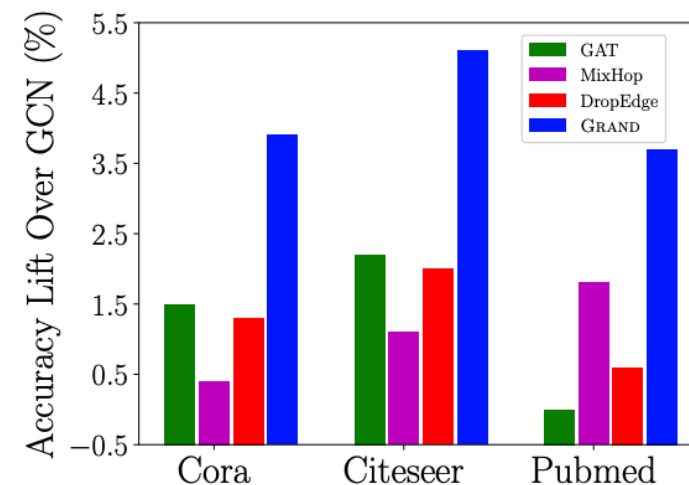
$$\mathcal{R}_{Do}^c(\mathbf{W}) = \frac{\delta}{1 - \delta} \sum_{h=0}^{d-1} \mathbf{W}_h^2 \sum_{j=0}^{n-1} \left[ \mathbf{X}_{jh}^2 \sum_{i=0}^{n-1} z_i^2 (1 - z_i)^2 (\overline{\mathbf{A}}_{ij})^2 \right]$$

- With Supervised Cross-Entropy Loss:

- Random propagation can enforce the consistency of the classification confidence between each node and its labeled multi-hop neighborhoods.

# Results

	Method	Cora	Citeseer	Pubmed
GCNs	GCN [19]	81.5	70.3	79.0
	GAT [32]	83.0±0.7	72.5±0.7	79.0±0.3
	APPNP [20]	83.8±0.3	71.6±0.5	79.7±0.3
	Graph U-Net [11]	84.4±0.6	73.2±0.5	79.6±0.2
	SGC [36]	81.0±0.0	71.9±0.1	78.9±0.0
	MixHop [1]	81.9±0.4	71.4±0.8	80.8±0.6
	GMNN [28]	83.7	72.9	81.8
	GraphNAS [12]	84.2±1.0	73.1±0.9	79.6±0.4
Sampling GCNs	GraphSAGE [16]	78.9±0.8	67.4±0.7	77.8±0.6
	FastGCN [7]	81.4±0.5	68.8±0.9	77.6±0.5
Regularization GCNs	VBAT [10]	83.6±0.5	74.0±0.6	79.9±0.4
	G <sup>3</sup> NN [24]	82.5±0.2	74.4±0.3	77.9±0.4
	GraphMix [33]	83.9±0.6	74.5±0.6	81.0±0.6
	DropEdge [29]	82.8	72.3	79.6
	<b>GRAND</b>	<b>85.4±0.4</b>	<b>75.4±0.4</b>	<b>82.7±0.6</b>



Instead of the marginal improvements by conventional GNN baselines over GCN, **GRAND** achieves *much more significant* performance lift in *all three datasets*!

- Feng et al. Graph Random Neural Networks for Semi-Supervised Learning on Graphs. <https://arxiv.org/abs/2005.11079>, 2020
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# Results

Table 5: Results on large datasets.

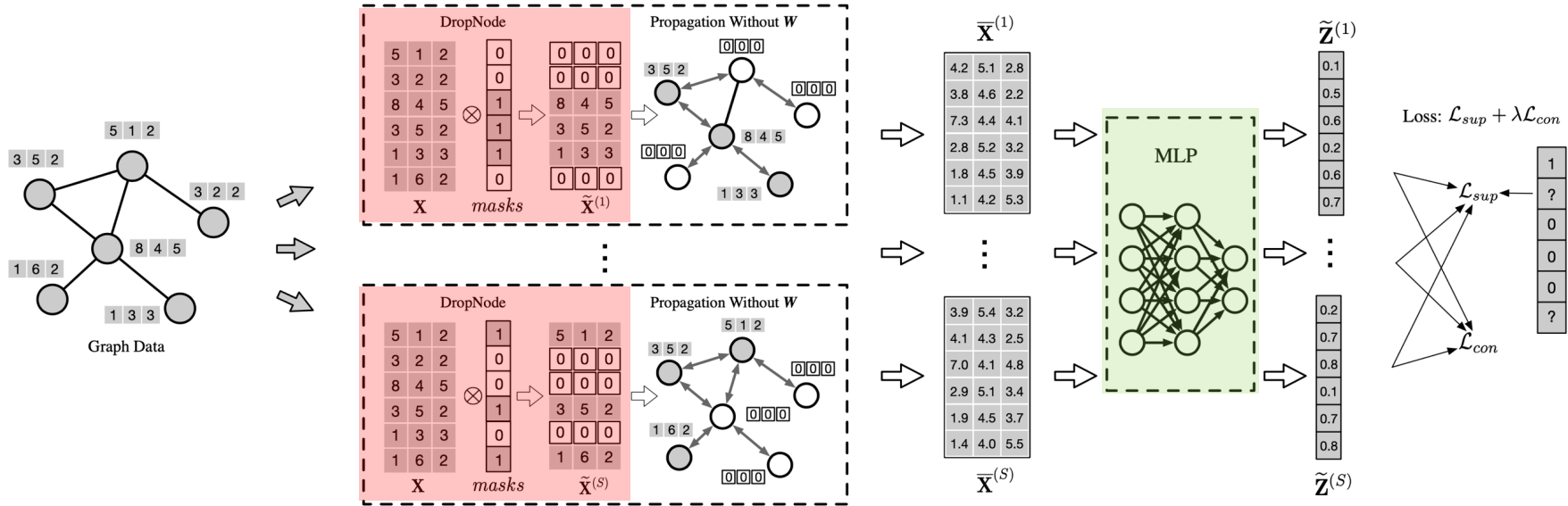
Method	Cora Full	Coauthor CS	Coauthor Physics	Amazon Computer	Amazon Photo	Citation CS
GCN	$62.2 \pm 0.6$	$91.1 \pm 0.5$	$92.8 \pm 1.0$	$82.6 \pm 2.4$	$91.2 \pm 1.2$	$49.9 \pm 2.0$
GAT	$51.9 \pm 1.5$	$90.5 \pm 0.6$	$92.5 \pm 0.9$	$78.0 \pm 19.0$	$85.7 \pm 20.3$	$49.6 \pm 1.7$
<b>GRAND</b>	<b><math>63.5 \pm 0.6</math></b>	<b><math>92.9 \pm 0.5</math></b>	<b><math>94.6 \pm 0.5</math></b>	<b><math>85.7 \pm 1.8</math></b>	<b><math>92.5 \pm 1.7</math></b>	<b><math>52.8 \pm 1.2</math></b>

More experiments on larger graph datasets

- Feng et al. Graph Random Neural Networks for Semi-Supervised Learning on Graphs. <https://arxiv.org/abs/2005.11079>, 2020
- Code & data for Grand: <https://github.com/Grand20/grand>



# Results



GRAND_dropout	84.9±0.4	75.0±0.3	81.7±1.0
GRAND_GCN	84.5±0.3	74.2±0.3	80.0±0.3
GRAND_GAT	84.3±0.4	73.2±0.4	79.2±0.6
<b>GRAND</b>	<b>85.4±0.4</b>	<b>75.4±0.4</b>	<b>82.7±0.6</b>

Evaluation of the design choices in GRAND

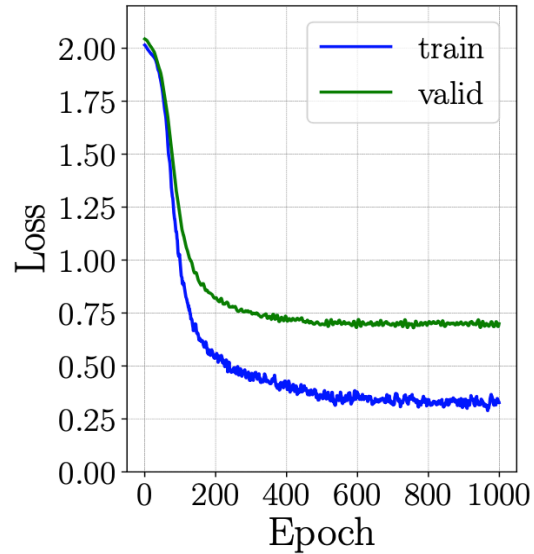
# Results

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w/o CR	84.4±0.5	73.1±0.6	80.9±0.8
w/o mDN	84.7±0.4	74.8±0.4	81.0±1.1
w/o sharpening	84.6±0.4	72.2±0.6	81.6±0.8
w/o CR & DN	83.2±0.5	70.3±0.6	78.5±1.4

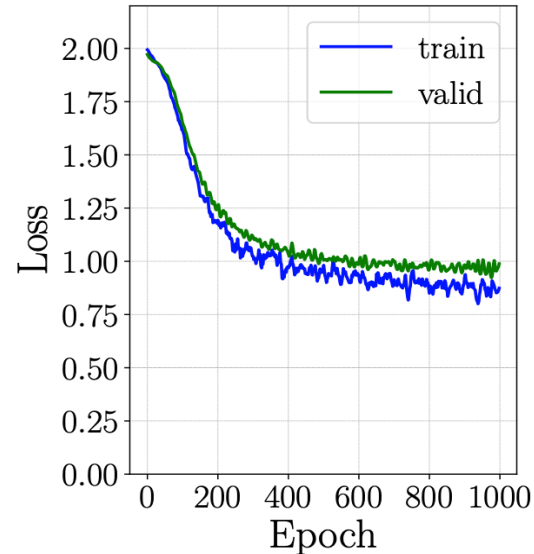
## Ablation Study

1. Each of the designed components contributes to the success of GRAND.
2. GRAND w/o consistency regularization outperforms almost *all 8 non-regularization based GCNs & DropEdge*

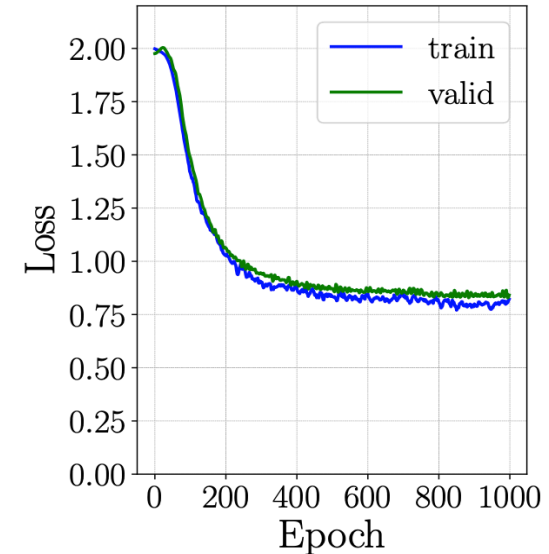
# Results



(a) Without CR and RP



(b) Without CR

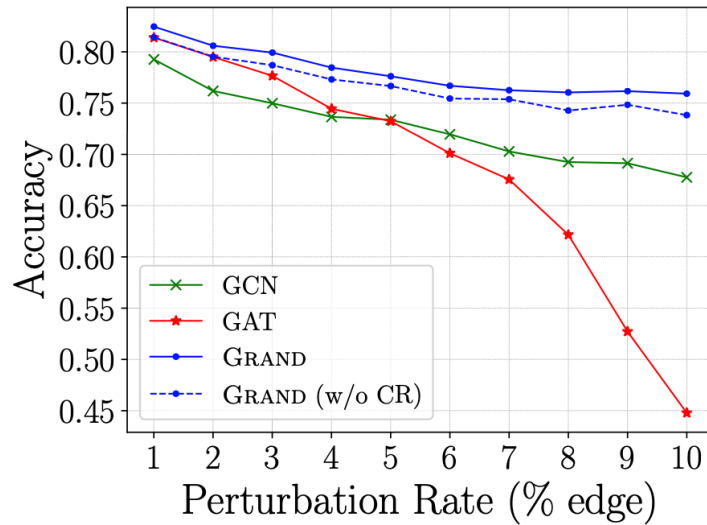


(c) GRAND

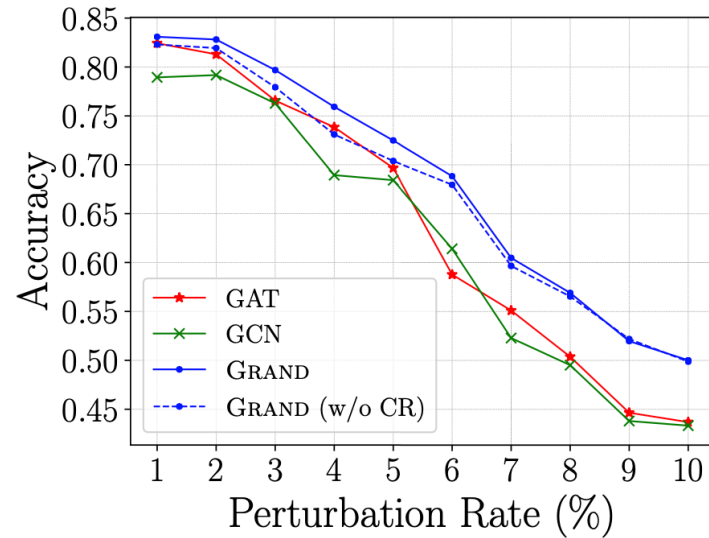
## Generalization

1. Both the random propagation and consistency regularization improve GRAND's generalization capability

# Results



(a) Random Attack

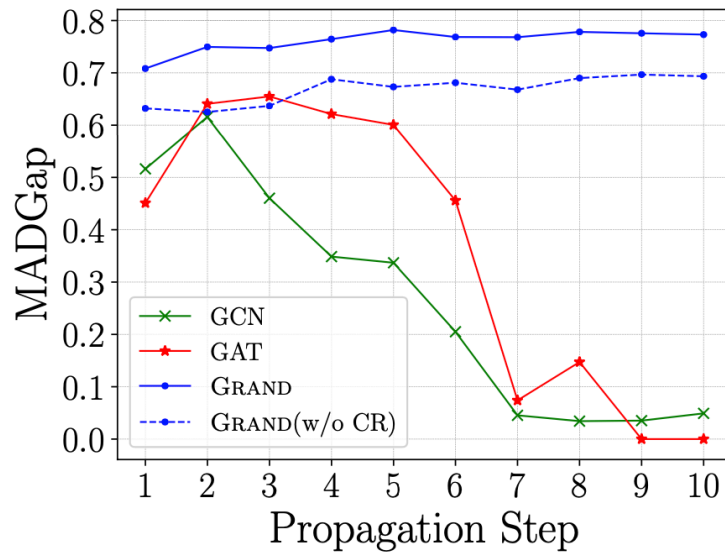


(b) Metattack

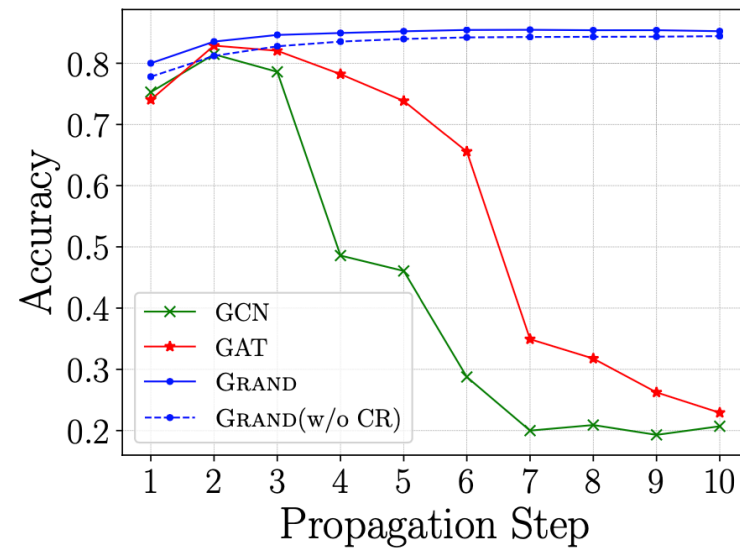
## Robustness

1. GRAND (with or w/o) consistency regularization is more robust than GCN and GAT.

# Results



(a) MADGap



(b) Classification Results

## Over-Smoothing

1. GRAND is very powerful to relieve over-smoothing, when GCN & GAT are very vulnerable to it



# Thanks!

Code & data for Grand: <https://github.com/Grand20/grand>